(8 pages) **Reg. No. :** ....

Code No. : 20292 E **Sub. Code : AMMA 51**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics – Core

LINEAR ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Which of the following is a subspace of a vector space R^3?
   (a) $W = \{(a, 0, 0)/a \in R\}$
   (b) $W = \{Ka, Kb, Kc/K \in R\}$
   (c) $W = \{(a, a+1, 0)/a \in R\}$
   (d) $W = \{(a, 0, b)/a, b \in R\}$

2. Let V be a vector space over a field F and W, a subspace of V. If $T : V \to \frac{V}{W}$ defined by $T(V) = W + V$ is a linear transformation, $\ker T =$ ————.
   (a) $\{0\}$ (b) V
   (c) $\{1\}$ (d) W

3. If $S = \{(2, 0)\}$ in $V_2(R)$ then $L(S) =$ ————.
   (a) $\{(x, 0)/x \in R\}$ (b) $\{(0, x)/x \in R\}$
   (c) $\{(0, 0)\}$ (d) $\{(0, 2)\}$

4. The vectors (a, b) and (c, d) are linearly dependent iff ————.
   (a) $ab - cd = 0$ (b) $ac - db = 0$
   (c) $ab - bc = 0$ (d) $ad - bc = 0$

5. $T : V_2(R) \to V_2(R)$ given by $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ with respect to the standard basis then the linear transformation is ————.
   (a) $T(a, b) = (a\sin\theta + b\cos\theta, -a\cos\theta + b\sin\theta)$
   (b) $T(a, b) = (a\cos\theta + b\sin\theta, -a\sin\theta + b\cos\theta)$
   (c) $T(a, b) = (-a\sin\theta + b\cos\theta, a\cos\theta + b\sin\theta)$
   (d) $T(a, b) = (-a\cos\theta + b\sin\theta, a\sin\theta + b\cos\theta)$

6. The norm of the vectors in $V_3(R)$ with standard inner product (3, –4, 0) is ————.
   (a) 3 (b) 0
   (c) 5 (d) –4

7. The rank of the matrix is $\begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & -3 \\ 6 & -1 & 1 \end{bmatrix}$ is ————.
   (a) 2 (b) 6
   (c) –2 (d) –3

8. If $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ then $|A| =$ ————.
   (a) 0 (b) 2
   (c) 4 (d) 1

9. For what value of k is 3 a characteristic root of $\begin{pmatrix} 3 & 1 & -1 \\ 3 & 5 & -k \\ 3 & k & -1 \end{pmatrix}$.
   (a) 5 (b) 2
   (c) –1 (d) 3

10. The eigen values of $\begin{pmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$ are
    (a) 3, 4, 1 (b) 3, 5, 3
    (c) 3, 0, 0 (d) 1, 1, 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) (i) Prove that the intersection of two subspaces of a vector space is a subspace.

    (ii) Prove that the union of two subspaces of a vector space need not be a subspace.

    Or

    (b) Let V be a vector space over a field F. A non-empty subset W of V is a subspace of V iff $u, v \in W$ and $\alpha, \beta \in F \Rightarrow \alpha u + \beta v \in W$.

[P.T.O.]

12. (a) Prove that any subspace of a linearly independent set is linearly independent.

Or

(b) Prove that $S = \{(1,0,0),(0,1,0),(1,1,1)\}$ is a basis for $V_3(R)$.

13. (a) Let V be the set of all continuous real valued functions defined on the closed interval $[0,1]$. Prove that V is a real inner product space with inner product defined by

$$\langle f, g\rangle = \int_0^1 f(t)\, g(t)\, dt$$

Or

(b) Let V be a finite dimensional inner product space. Let W be a subspace of V. Prove that $(W^{\perp})^{\perp} = W$.

14. (a) Show that the non-singular matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ satisfies the equation $A^2 - 2A - 5I = 0$. Hence evaluate A^{-1}.

Or

(b) State and prove Cayley-Hamilton theorem.

15. (a) Let f be the bilinear form defined by $V_2(R)$ by $f(x, y) = x_1 y_1 + x_2 y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of f w.r.t. the basis $\{(1,1),(1,2)\}$.

Or

(b) Find the characteristic roots of the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let V and W be vector spaces over a field F. Let $L(V, W)$ represent the set of all linear transformations from V to W. Then $L(V, W)$ itself is a vector space over F under addition and scalar multiplication defined by $(f + g)(v) = f(v) + g(v)$ and $(\alpha f)(v) = \alpha f(v)$.

Or

(b) State and prove Fundamental theorem of Homomorphism.

17. (a) Let V be a finite dimensional vector space over a field F. Let W be a subspace of V. Prove that

(i) $\dim W \le \dim V$

(ii) $\dim \dfrac{V}{W} = \dim V - \dim W$.

Or

(b) Let V be a vector space over a field F. Let $S, T \subseteq V$, then prove that

(i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

(ii) $L(S \cup T) = L(S) + L(T)$

(iii) $L(S) = S \Leftrightarrow S$ is a subspace of V.

18. (a) Let V be the vector space of polynomials with inner product given by $\langle f, g\rangle = \int_0^1 f(t)\, g(t)\, dt$. Let $f(t) = t + 2$ and $g(t) = t^2 - 2t - 3$. Find

(i) $\langle f, g\rangle$

(ii) $\|f\|$.

Or

(b) Show that every finite dimensional inner product space has an orthonormal basis.

19. (a) Verify whether the following system of equations is consistent. If it is consistent find the solution

$$\begin{aligned} x - 4y - 3z &= -16 \\ 4x - y + 6z &= 16 \\ 2x + 7y + 12z &= 48 \\ 5x - 5y + 3z &= 0. \end{aligned}$$

Or

(b) Find the inverse of the matrix $\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ using Cayley-Hamilton theorem.

20. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

Or

(b) Reduce the quadratic form $2x_1x_2 - x_1x_3 + x_1x_4 - x_2x_3 + x_2x_4 - 2x_3x_4$ to the diagonal form using Lagrange's method.

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